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Magnetic fields and the cosmic microwave background

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Abstract

Observations of the high degree of isotropy of the cosmic microwave background are commonly believed to indicate that the Universe is 'almost' Friedmann–Lemaître–Robertson–Walker (at least since the time of last scattering). Theoretical support for this belief comes from the so-called Ehlers–Geren–Sachs theorem. We show that a generalization of this theorem rules out any strong magnetic fields in the Universe. Our theoretical result is model-independent and includes the case of inhomogeneous magnetic fields, complementing previous results. We thus prove that cosmic microwave background observations severely constrain all types of primordial and protogalactic magnetic fields in the universe.

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Magnetic fields seem to be abundant in the universe, and it is an open and important question as to whether the origin of these fields are either primordial (i.e. originating in the early universe and already present at the onset of structure formation), or protogalactic (i.e. generated by battery mechanisms during the initial stages of structure formation). One way to distinguish between these two possibilities would be to detect or rule out the presence of fields coherent on cosmological scales during recombination via their imprint on the cosmic microwave background (CMB) radiation. Dynamically significant magnetic fields present during recombination must be primordial. Indeed, any *large-scale* primordial magnetic field with a strength comparable to that inferred from the lowest measured intergalactic fields and close to the observational upper limits via Faraday rotation measurements [1] may well be of cosmological origin. The detected magnetic field strengths in high-redshift galaxies [2] and in damped Lyman alpha clouds [3] are consistent with the observed Faraday rotation measurements. Primordial magnetic fields can be created in the early universe through phase transitions or via inflation. However, even the invocation of protogalactic dynamos to explain the magnitude of the field involves many uncertain assumptions and still requires a small primordial (pregalactic) seed field [4]. Any seed magnetic field is then amplified adiabatically during gravitational collapse. Hence the possibility of a primordial field merits serious consideration.

It is consequently of primary interest to determine the effects these magnetic fields have on the CMB and, in particular, to consider the limits on any large-scale primordial field from the CMB isotropy measurements. Indeed, the CMB measurements may lead to severe constraints on *homogeneous* magnetic fields in particular classes of cosmological models [5]. Here we demonstrate quite generally that severe limits may be put on *any* large-scale magnetic field—be it homogeneous, as considered before, or inhomogeneous—from consideration of the high isotropy of the CMB. That is, we show that any universe in which its observers measure an exactly isotropic CMB cannot have a large-scale magnetic field. We show this by demonstrating that the magnetic field evolves in a manner which is irreconcilable with the spacetime geometry required by the (exact) isotropy of the CMB. Contrary to popular belief, this has not been done before.

We wish to take a form of the Copernican principle—which is the fundamental assumption in most of cosmology—combined with our well established knowledge of the high isotropy of the CMB as our starting point. Thus we wish to consider the class of spacetimes which allow *every* observer to see an isotropic CMB, and determine whether any of these are compatible with a large-scale magnetic field. Depending on additional assumptions that can be made, a surprising amount can be inferred from this starting point; for example, if the matter in the spacetime is purely baryonic cold dark matter ('dust') with a contribution from radiation then the resulting spacetime must be Friedmann–Lemaître–Robertson–Walker (FLRW). This profound result is known as the Ehlers–Geren–Sachs theorem (EGS, [6, 7]) and underpins the standard model of cosmology, not to mention many of the results of CMB physics. However, the Universe is not made up only of baryons but also dark matter, most of which could well be non-baryonic [8]. In addition, in light of the supernovae Ia data [9], there could be a large contribution from a dynamic scalar field (e.g. quintessence [10]), and there is no reason to believe that this would be homogeneous. Thus the EGS theorem is not the final word on the validity of the homogeneity of the standard model, and recent work has investigated certain generalizations of the EGS theorem [11–15] to inhomogeneous models. Most importantly for this work is the role a magnetic field would play in relation to these theorems.

In its most general form, the 'isotropic radiation field' theorem identifies spacetimes in which all observers see an 'isotropic radiation field'. This may be derived from the Einstein–Boltzmann equations for photons in a curved spacetime [7, 16]. It is then easy to show from the multipole expansions of [16] that in a spacetime in which all observers on some timelike congruence u^a see an exactly isotropic radiation field, then this velocity field has two important properties [6, 11]: first, the expansion θ and acceleration $\dot{u}_a$ (the motion of the observers under non-gravitational forces) of these observers are related by

$$\dot{u}_a = \widetilde{\nabla}_a Q, \qquad \theta = 3\dot{Q}, \tag{1}$$

where Q is related to the energy density of the radiation field. Here '$\widetilde{\nabla}_a$' refers to the gradient in the instantaneous rest space of the observers (a generalization of the familiar 'grad' of Euclidean three-dimensional vector calculus), and '$\dot{}$' is a proper time derivative along the fluid flow. Second, the congruence must be shear-free, which means that the congruence cannot have any distortion (note that this condition is *not* satisfied in the spacetimes of [5]). In many cases of physical interest it can then be proven that these observers have zero rotation[1], a condition supported in part by the isotropy of number counts [17]. For simplicity of exposition

[1] For example, if part of the matter consists of a conserved comoving barotropic perfect fluid other than radiation, or for geodesic motion with any matter source, it follows from (1) that the expansion or the rotation must be zero. (For a conserved barotropic perfect fluid, we have $\dot{u}_a = \widetilde{\nabla}_a\phi$, and $p'\theta = \dot{\phi}$, where $\phi \equiv -\int \mathrm{d}p/(\mu(p) + p)$, and $p' = \mathrm{d}p/\mathrm{d}\mu$; so, $\eta_{abc}\widetilde{\nabla}^b\widetilde{\nabla}^c(Q - \phi) = 2(\frac{1}{3} - p')\theta\omega_a = 0$. For geodesic motion, $\eta_{abc}\widetilde{\nabla}^b\widetilde{\nabla}^c Q = \frac{2}{3}\theta\omega_a = 0$ [11].)

we shall examine only the irrotational case here. The general case of non-zero rotation is complicated, and will be considered elsewhere [14].

Now, if we have a pure magnetic field B^a, it will contribute an anisotropic pressure[2], $\Pi_{ab} = -B_{\langle a}B_{b\rangle}$ to the energy–momentum tensor of the spacetime, as well as an energy density $\mu_B = \frac{1}{2}B^2$, and an isotropic pressure, $p_B = \frac{1}{3}\mu_B$ [16, 19, 20]. The energy of the magnetic field must be conserved, as required by one of the Einstein–Maxwell equations (the induction equation), which is an evolution equation for the magnetic field vector;

$$\dot{B}_{\langle a\rangle} = -\tfrac{2}{3}\theta B_a. \tag{2}$$

This implies that the anisotropic pressure evolves as [20]

$$\dot{\Pi}_{\langle ab\rangle} = -\tfrac{4}{3}\theta \Pi_{ab}; \tag{3}$$

it is this evolution equation—which is a direct consequence of Maxwell's equations—which is inconsistent with Einstein's field equations when an isotropic radiation field is present, as we shall now show.

In the usual 1 + 3 covariant approach, Einstein's field equations are broken into a set of evolution and constraint equations [16], which we specialize to our particular case, to determine whether the magnetic field may form part of the source. The evolution equation for the shear, in our case, becomes an algebraic equation between the anisotropic pressures, Π_{ab}, and the electric part of the Weyl tensor, E_{ab}, which governs tidal forces and gravitational waves:

$$E_{ab} - \tfrac{1}{2}\Pi_{ab} = A_{ab}, \tag{4}$$

where $A_{ab} = \widetilde{\nabla}_{\langle a}\dot{u}_{b\rangle} + \dot{u}_{\langle a}\dot{u}_{b\rangle}$ is a contribution from acceleration terms. The electric Weyl tensor must satisfy the evolution equation

$$\dot{E}_{\langle ab\rangle} + \theta E_{ab} = -\tfrac{1}{2}\left(\dot{\Pi}_{\langle ab\rangle} + \tfrac{1}{3}\theta\Pi_{ab}\right). \tag{5}$$

(These equations may be found in, for example, [16, 18] for a general spacetime.) It may be shown that the acceleration terms A_{ab} evolve as

$$\dot{A}_{\langle ab\rangle} = -\tfrac{1}{3}\theta A_{ab}. \tag{6}$$

(This may be shown using (1), the Ricci identities, and equations (5) and (14) from [18]: i.e. the fact that there is no energy flux from the magnetic field and the matter implies that the expansion is homogeneous when the rotation is zero, which in turn implies that the acceleration vector evolves parallel to the velocity vector.) Now, combining (4)–(6) yields the following evolution equation for the anisotropic pressure:

$$\dot{\Pi}_{\langle ab\rangle} = -\tfrac{2}{3}\theta\Pi_{ab} - \tfrac{2}{3}\theta A_{ab}. \tag{7}$$

This is the general evolution equation for the anisotropic pressure for an irrotational spacetime without energy flux which allows an isotropic radiation field. It is now easy to see that a magnetic field is inconsistent with this evolution rate: a magnetic field in these spacetimes must satisfy (3), which, upon substitution into (7), and using (6) implies that

$$\theta\Pi_{ab} = 0; \tag{8}$$

i.e. the spacetime must be static (which is not relevant to cosmology), or the magnetic field must vanish (in which case the inhomogeneous models will be those found in [11], and are

[2] Angled brackets denote the 'projected, symmetric, trace-free' part of a tensor, where the projection is into the observers' instantaneous rest space, and are also used to denote the projected symmetric and trace-free part of time derivatives of spatial tensors (i.e. the angled brackets are applied after the time derivative) [16, 18].

not FLRW in general). Note that we have made no assumptions on the other matter present, other than it should have a perfect fluid form; it need not be barotropic, or homogeneous (both assumed in [5]), or geodesic; it may also consist of many different components, for example, a perfect fluid and a scalar field (e.g. quintessence). No additional assumption on the spacetime geometry has been made; this result is consequently model-independent. *Thus we see that, in quite general terms, a magnetic field in the Universe is not compatible with an exactly isotropic CMB.*

However, it should be noted that we chose the magnetic field to be 'pure' in the frame of the radiation, i.e. 'frozen into' the matter; if it were pure in another frame (i.e. with respect to some other observers), the energy flux (the Poynting vector arising from the electric field due to motion through the magnetic field [21]) would not be zero and the result would not follow directly from the above argument. Also, in principle, models which contain other anisotropic matter stresses need to be considered separately (e.g. if there were anisotropic pressures from some other source which happened to cancel with those of the magnetic field, then this result would not necessarily apply from the above calculation).

Of course, this result is a theoretical result based on perfect isotropy of the CMB, and this magnetic-EGS theorem is not *directly* applicable to the real Universe, since the CMB temperature is not *exactly* isotropic. The original EGS theorem has been generalized to the 'almost EGS theorem' [22], which states that if all fundamental observers measure the CMB temperature to be almost isotropic during some time interval in an expanding dust[3] universe, then the universe is described by an almost FLRW model during this time interval. Therefore, there will be an accompanying approximate result (an 'almost' magnetic result) from which the observed isotropy of the CMB will lead to severe constraints on the magnetic field. This may be done by simply following the above proof, but now including terms which are zero here but keeping them small: thus we may derive a specific value for the maximum strength of any magnetic field from our knowledge of the CMB anisotropies[4]. Indeed, Barrow *et al* [5] have shown that the constraints from the CMB isotropy measurements may provide very strong limits on the strength of a homogeneous component of a primordial magnetic field, and stronger than those imposed by primordial nucleosynthesis constraints. However, the results of [5] are not generic and are model dependent (see [24] for details[5]); furthermore, they only apply to homogeneous magnetic fields. In particular, as the models in [5] have shear, it is not clear whether the derived constraints are really characteristic of the presence of the magnetic field[6]. Hence our theoretical result (which is *not* model dependent) complements their results very nicely and provides the necessary theoretical background to such limits. Indeed, a simple calculation shows that the constraints on magnetic fields from this analysis are of a similar order of magnitude obtained in [5], but an accurate limit requires a separate detailed analysis [26]. We have thus demonstrated that *severe constraints may be placed on magnetic fields in the Universe*

[3] Note that this assumption is crucial to the theorem [13, 23].

[4] There will, however, be some additional assumptions required in such a calculation [22]: for example, in an exact result such as ours, a quantity which is zero (such as the shear) will have zero derivatives; whereas, in the 'almost' case where a quantity is small, it may not necessarily have small derivatives. These terms must be kept small by assumption in order to complete the calculation.

[5] In particular, we note that in general Bianchi VII_h models, considered in [5], *cannot* have a magnetic field as a source [25].

[6] The authors consider Bianchi VII_h models, which are homogeneous flat spacetimes with shear. Therefore, the models will have an anisotropic CMB, regardless of how one chooses the matter content, as a consequence of the isotropic radiation field theorem, because it prohibits spacetimes with shear. In their work, the matter was chosen to be a magnetic field and hence was related to the shear in a particular fashion. It is not clear to what extent the anisotropy of the CMB in these models constrain the magnetic field through its particular coupling to the shear, and to what extent it restricts magnetic fields in general.

as a consequence of the extremely high isotropy of the CMB. In particular, our results also apply to inhomogeneous magnetic fields, and we note that any inflationary scenario leading to significant large-scale primordial magnetic fields would presumably result in magnetic inhomogeneities.

So far we have considered how constraints on *large-scale* magnetic fields may be derived from the CMB anisotropies. We have not, however, given consideration to other types of magnetic field which may be present in the Universe. Specifically, 'tangled' magnetic fields—i.e. strong magnetic fields which are inhomogeneous on small scales, but 'average' to zero on large scales—may also be constrained in a similar way. For example, consider a bundle of light rays coming from the CMB surface to us of some angular size. We have seen that a magnetic field will distort an isotropic radiation field, so that if our bundle were to pass through a strong small-scale magnetic field, it would produce temperature anisotropies in the CMB multipoles corresponding to the size of the tangled field. This may be used to place limits on the strength and size of these inhomogeneous small-scale fields [26]. With the coming launch of the next-generation MAP and Planck satellites to measure the high-order multipoles of the CMB spectrum, we may be able to place stringent limits on these types of fields.

We have considered how constraints on magnetic fields may be derived from the temperature anisotropies of the CMB. What, then, would be the implication of discovering, by some other means, a large-scale magnetic field in the Universe? It would follow immediately that the real Universe does not satisfy some or all of the assumptions used in this theorem. Most interestingly, it could be indicating that all observers in the Universe do not measure such high isotropy of the CMB; that is, that the Copernican principle does not hold. Thus we see an unexpected route for a physical test of the Copernican principle—looking for magnetic fields.

All of the discussion here has been confined to general relativity. It should be noted that the situation might be different in alternative theories of gravity. For example, in scalar–tensor theories of gravity and low-energy effective theories derived from string theory, the high isotropy of the CMB when combined with the Copernican principle implies that the Universe is isotropic and homogeneous in the case of geodesic matter [15]. However, in string theory the electromagnetic field is coupled to the dilaton (unlike in general relativity in which the electromagnetic field is governed by the conformally invariant Einstein–Maxwell equations), and so the vacuum fluctuations of the electromagnetic field can be significantly amplified by accelerated growth of the dilaton in the pre-big-bang phase; in this case the primordial magnetic field might be strong enough to seed galactic dynamo effects and explain the origin of cosmic magnetic fields observed on galactic and intergalactic scales [27]. Therefore, the CMB constraints will probably be even more restrictive in alternative theories of gravity.

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