On mathematics education in SA and the relevance of popularising mathematics

working paper

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Abstract

A collection of short essays is given on circumstances surrounding mathematics and science education in SA and on the popularisation of mathematics. The aim is to promote interest in mathematics, science and scientific enquiry, and to place mathematics in the context of social development. Subsections may be read independently of each other.

Mathematics and Science are part of human existence and development, and, therefore, our collective culture. The pursuit of such knowledge should be encouraged vigorously at all levels of education.

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1 On the status of science education in SA and developing countries (1999)

In South Africa at the moment there is general recognition that general education, including literacy, numeracy and scientific awareness is not in keeping with average levels attained in certain other countries, including highly industrialised and technologically developed countries. To illustrate some secondary effects attributable to the educational crisis in SA, and other countries with similar levels of development, consider some indicators[1] :

- (1995) 9 million[2] adults illiterate in SA.
- SA scholars perform poorest out of 41 countries participating in the Mathematics Achievmentment survey at uppergrade level in TIMMS, 1994-95; and also attain the lowest average out of 21 countries participating in the Mathematics and Science Literacy survey at final year level in TIMMS, 1995-96.
- There is a shortage of between 4000 and 1200 math and science teachers in SA (1996).
- In SA scientists and engineers stand at 3.3 per 1000 of the population compared with 21.5 per 1000 and 71.1 per 1000 in US and Japan. respectively
- Sub-Saharan Africa [excluding SA] has one internet user per 1500 compared to the 1 in 4 in Europe and North America.
- 20% of humanity in industrialised countries account for 90% of the production of scientific knowledge.
- About US$ 470 billion is spent on R&D and only 10% of this comes from the South.

[1] the numbers presented here are based on research in 1999
[2] this figure is unconfirmed

There is, however, commitment to not only increasing basic literacy and numeracy but also scientific literacy and education. Several universities and other tertiary education institutions across the country have taken various initiatives with regard to math and science education and popularisation. Some projects at the University of Cape Town include:

- Alternative Admission to university under the Alternative Admissions Research project. This is intended to serve particularly students from academically disadvantaged backgrounds. Students take specially designed university entrance tests to assess their ability.
- Academic Development and Science Foundation Programmes. These serve as bridging programmes for first-year students from disadvantaged backgrounds. The academic load of these programmes is less in the first year, while there is additional tutoring support.
- A Mathematics Education Project to provide in-service training, workshops, seminars and resources to primary and secondary level mathematics teachers.
- A Numeracy Centre to ensure that every university student has access to some form mathematics education (this is particularly relevant in the SA context).

The National Research Foundation (NRF) is committed to the sustaining and developing science and technology. Government too has made repeated statements of commitment [3]: the recently appointed Minister of Education has pledged to improve the situation in schools by, amongst other things, improved teacher training and in-service training; at the Third World Organisation for Women in Science conference held in Cape Town in Feb 1999, the head of state pledged support to the interests of women in science and the general promotion of science.

There is still, however, work to do: the problems faced are not isolated from the other basic problems in SA including a housing crisis, critical unemployment, and the vast sprawl of squatter areas around all the major cities. Children do not learn well when they go to school on empty stomachs or when they struggle through winters without adequate protection from the cold and wet.

Globally there remains genuine concern that the range of benefits occurring as a result of scientific and technological advance do not to serve the needs of the marginalised poor, and those without formal education.

[3] see the preamble of the new Parliamentary Bill on Higher Education

Fortunately, information and communication technology provide the means to the interaction needed for research, education and social development. Furthermore, the gradual accumulation of experience and the documented projects become valuable resources for ideas and approaches to problems. With the range of projects and initiatives that have been implemented within recent years in SA, it becomes useful and necessary to work collaboratively.

Most industrialised and technologically advanced countries have seen an unprecedented proliferation in tertiary education. This phenomenon has been dubbed the *democratisation of education*. To some extent this process is taking place in South Africa. Thus, universities in SA now are expected not only to provide access to general higher education and professional training, but also to compensate for decades of substandard primary and secondary education. There are implementations of innovative ways of making university education accessible to as wide a student population as possible. Short term success is limited, and long-term rewards cannot be measured yet.

It is well known that a substantial portion of learning takes place via peer interaction. Bearing this in mind, and the fact that more and more students are of the first generation in their families to receive secondary and/or tertiary education, or even primary education, it becomes critical to find means to transfer both formal and informal or tacit knowledge to scholars and students.

> *The paramount importance of systemized education has long since been recognized. For the future of society, it is intimately bound up with, and to a great extent dependent upon the way, the youth are prepared for the task of maintaining that society. Any system of education, therefore, is concerned not only with the teaching of certain skills and professions, but with inculcating certain ideas which are basic to the society and necessary for it's perpetuation.* I.B. Tabata

Mathematics and science are basic to human development and the functioning of society. The rich tradition of knowledge and understanding which they offer belongs to everybody.

2 A brief history of past education policy in South Africa

> *Even if we harden our hearts to the shame and misery experienced by the victims, the cost of of illiteracy [and innumeracy] to everyone is severe - the cost of medical expenses and hospitalisation, the cost of crime and prisons, the cost of special education, the cost in lost productivity and in potentially brilliant minds who could help solve the dilemmas besetting us.* Carl Sagan

The South African education system is in a partial crisis as a result of a deficient national education system implemented under its previous system of government. Up until 1994 the educational system was still formally part of a complex legislated system of Apartheid or segregation. The Apartheid system was an attempt to formalise the misguided notions of racial superiority and inferiority. The intention and effect of this was the securing of the ownership of various and extensive agricultural and mineral resources with the cheap labour for development that would benefit a select minority of some 13%. The Report of the Departmental Committee on Native Education (1935-6) sums up the policy:

> *The Education of the White child prepares him for life in a dominant society and the Education of the Black child for a subordinate society...The limits [of Native Education] form part of the social and economic structure of the country.*

By the early 1950's, the ruling government of the Union of SA crowned decades of legislation, which had stripped Africans of political and economic rights, with an Education policy designed to keep them in subservience. To quote Hendrik Vervoed who went on to become President:

> *My Department's policy*[4] *is that education should stand with both feet in the Reserves*[5] *and have its roots in the spirit of being Bantu society*[6] *... There is no place for him [the African] in the European community*[7] *above the level of certain forms of labour.*

[4]The Department of Native Affairs of that time.

[5]The Reserves were the only 12% of SA territory where Africans could own land and livestock.

[6]African's themselves abhorred being called *Bantu*. This was name given to them by the descendants of the Dutch Boers. The Boers called themselves *Afrikaners*, which has the English translation *African*, in order to reflect their own claim to the land.

[7]European was the term used to describe all *White* South Africans.

This policy systematically ensured that the original Africans were prevented from obtaining an education in keeping with the advances of the twentieth century while they built such an environment for a post-colonial minority. The system was in keeping with the philosophies that supported slavery as echoed in the 19th century by an American slave-owner:

> *A nigger should know nothing but to obey his master - to do as he is told to do. Learning would spoil the best nigger in the world. Now if you teach that nigger how to read, there would be no keeping him. It would forever unfit him to be a slave.*

The education system designed and implemented by the orchestrators of Apartheid furthermore had to entrench the notions of superiority and national pride amongst Whites and make this sanctified as something God-given:

> *We believe that these principles [Christian and National] should come to full fruition in the education of our children so that these two principles permeate the whole school in regard to it's spirit, aim, curricula, method, discipline, staff, general organization and all it's activities.*

So stated the *Christian National Education* (CNO) policy adopted by the leaders of Apartheid.

> *Native education should be based on the principles of trusteeship, non-equality and segregation,*

it continues. Notably such policies was not unique in its some of its goals as a national education system. The CNO policies had in fact stemmed from preparations which began before the second world war, and were modelled on some of the policies of the education system of Nazi Germany:

> *National Socialist ideology is to be a sacred foundation... It is a holy unit that must be accepted as a holy unit. It must be taught by teachers who fully comprehend the true meaning of our sacred doctrines.*

These words by a Nazi minister reflected equally well the sentiments of some of the creators of the former South African educational policy. Fortunately, the wholesale attempt at brainwashing failed. It was rejected even by children, as was reflected in the 1976 uprisings of scholars and students, and the continuous student protests thereafter. On the policies of the Nazi education system, a German once professor said:

> *If one destroys a power plant, it is dark at once, but if one destroys the Universities, it is dark fifty years hence.*

It is this sort of 'darkness' affecting all the levels of education in SA that must be addressed. In light of the significance of mathematics to education in general, it is not surprising that the former education system had deemed training in mathematics irrelevant. Vervoed had claimed in Parliament when introducing the Bantu Education Bill:

> *What is the use of teaching the Bantu child mathematics when it cannot use it in practice...That is quite absurd.*

Although, Apartheid education did extend schooling amongst Africans, it kept spending to a minimum. Thus we are left with shortages in school facilities, material resources, and, in particular, a shortage of skilled mathematics and science teachers. As in any other country, South Africa is still faced with the immense task of adequately educating its population.

There is now growing support for the formulation of policies to reverse the shortages. We need to be optimistic. Other countries have 'bridged the knowledge gap'. We may consider some of the lessons, of success and failure, provided for example, by some of the countries in East Asia later this century, and by the Soviet Union earlier on. There we may see how the appreciation for literacy, numeracy, knowledge, education, science and technology brought major advances and changes in attitudes and ways of thinking. As the mathematician Mark Kac writes,

> *Education cannot be based on being frightened by the immensity of knowledge but on the promise of insight which comes from the mastery of basic principles. To deny this is to deny every lesson of our history and to reject much of which has shaped our civilization.*

3 Popularising Mathematics, Science and Scientific Enquiry

3.1 How to promote mathematics

Interest in mathematics and science can be encouraged by providing exposure to:

- *interesting problems and beautiful theories* which usually have equally interesting histories, such as those found in the development of non-Euclidean geometries, modern abstract algebra, topology, and analysis,
- *exciting phenomena* which are encoded in the language of mathematics, such as dynamical systems, complexity and chaos theory which are really about the understanding of deeper order and patterns.

Alternatively the study of mathematics can be encouraged via applications which use mathematical techniques such as those found in

- the *scientific models* and theories in physics, cosmology and biology, and,
- *technological, environmental and economic solutions* e.g.problem solving in engineering, information processing and communication, resource management and development, and forecasting problems,
- *computing, simulation and artificial intelligence research.*

Grouping the above examples of mathematics into two camps, pure and applied, we may recall the words:

> *The two great streams of mathematical creativity are a tribute to the universality of human genius.* Mark Kac

It is critical for our collective advance that the body of knowledge being built up be shared amongst as many as possible. Through new and ancient insights and applications, mathematics and science have direct and indirect, but far-reaching, effect on the lives of all.

3.2 A sales pitch: mathematics and science for everyone and everything

The challenge of promoting mathematics is to find ways explain *why* mathematics should be studied. To promote music, one plays music, to promote art one shows pictures. To promote math one should *show the math*. However, this can demand the use of specialised language and symbols. Though it is not as convincing, we can quote to the words of mathematicians and scientists :

> *Mathematics is a vast adventure in ideas; its history reflects some of the noblest thoughts of countless generations.* D.J.Struik
>
> *Science is probably the most natural thing we as humans do - mathematics is an expression of this universality.* Carl Sagan

However, we are still faced with misconceptions that science is something only comprehensible to strange sorts who talk even stranger languages, or that mathematics is enjoyed by superior beings or maladjusted geeks. In fact, study and research in mathematics, pure or applied, merely demands interest and discipline, and is rewarding and personally satisfying. Thus, it is comparable to any serious pursuit, whether, music, ballet, sport or how to live one's life. And just as with any other serious pursuit, it requires courage when faced with challenge.

For those who make their way past the negative labelling mistakenly associated with an interest in science, and those who overcome the fear of a challenge to grapple with new ideas, the mathematics and science offer new understandings of the future and past of human civilisation.

- We have been able to *partially control the environment* and shape it to our needs, by building dams for the continuous supply of water, or by supplying energy with extensive electrification systems.
- Scientific techniques are able to remove guesswork and *provide predictions*, such as hurricane warnings, or the sex of a baby before birth.
- Seemingly unintelligible problems have found *clearer description*, and have become somewhat demystified with scientific experimentation and analysis, such as questions on the origin of life on earth.
- Mathematics and science have also humbled us and helped us understand our collective place in the greater universe - we know that the earth revolves around the sun which is a star amongst other stars in a galaxy amongst galaxies.

It is now the case that every domain of research applies some mathematics, and appeals to scientific techniques, for validity. Thus, Mathematics, science and technology have always been, and will continue to be deeply linked to human history.

3.3 What are Mathematics and Science?

Precise definitions of both *Mathematics* and *Science* can be subtle. In fact, sometimes descriptions are debated. However, there are generally accepted views of what the subjects are and it is important that those who are affected by, or interested in the development or use of mathematics or science (i.e. everyone), be informed of what these subjects are about. Furthermore, anyone should be allowed to form opinions on matters which affect them. Ideally, these should be as much a part of daily conversation as politics and sport are.

Mathematics is not just about counting and numbers, even though that is where it finds its origins. Nor is it just about calculation. It is about the *abstraction of ideas*. The word abstract here is taken in the sense of *identifying essential qualities from the full context for simplification and exactness*. We may consider: continuity, distance, size, shape, space, or collections of things which have common properties. These ideas are given more precise meaning in mathematics to prevent ambiguity, confusion or contradiction. They are translated into symbolic notation for simplicity, just like the letters of the alphabet serve as a code. Science is very much like mathematics in the sense that it tries to abstract essential information or properties of phenomena in order to gain deeper understanding. Special terminology gets introduced in order to clarify and discuss the things or ideas under investigation. By logical manipulation and rational investigation of ideas we are able to:

- state and prove (or corroborate) conjectures or theorems about abstract ideas, and how these ideas interrelate,
- develop theories about abstract concepts,
- solve problems based on information abstracted from problems, or
- gain insight into the workings of reality via abstract descriptions, models and theories about reality.

In general, through both mathematics and science, we are able to gain deeper insight into ourselves, our minds, society, the world the universe, and human society.

Just as mathematics and science may inform and change the way we think, thought in mathematics and science has changed over the centuries. Mathematics offers notions of truth and falsity via proof and calculation. However, the meanings of valid proof or good calculation have changed and evolved. Conclusions are based on facts. We may debate about what ‘facts’ are, how facts are to be viewed, and which methods of reasoning or logic are acceptable to derive conclusions. Arguments or proofs used hundreds of years ago may not be accepted today. For example, Greek mathematics or science differed from the mathematics and science of the Christian Middle Ages, which in turn differed from those of the Renaissance era. Furthermore, it has become widely accepted that there are no absolute truths. With this, comes the realisation that there are no absolute methods of how to do mathematics or science. There is no general 'follow steps A, B, C, D, etc, and you will get results.' Instead, mathematical and problem solving skills are developed by thought and practice. With applications, it is not always the case that we can fully determine an outcome, but we are able determine some consequences of action or assumption.

We are, however, able to say a few more things about the nature of scientific and mathematical enquiry today. Whether pure, or applied, mathematical and scientific activity includes the posing of questions, drawing conclusions by rational deduction, experimentation and observation, classification and measurement, the use of apparatus and equipment, and the use of language and terminology. Though the questions, answers, experiments, classifications, equipment and terminology may change, we've come to appreciate that mathematical and scientific activity should always demand free and sceptical thinking.

> *...at the heart of science is an essential balance between two seemingly contradictory attitudes - an openness to new ideas, no matter how bizarre or counterintuitive, and the most ruthlessly sceptical scrutiny of ideas...The collective enterprise of creative thinking and sceptical thinking, working together, keeps the field on track.* Carl Sagan

With this, we realise that we are forced to understand the world more as it is, rather than see what we choose. Because there should be no sacred truths, criticism and questioning is essential to the practice of math and science. This ensures that ideas are tested and probed.

> *If mathematics [or science] is to rejuvenate itself and break exciting new ground, it will have to allow for the exploration of new ideas and techniques which, in their creative phase, are likely to be as dubious as in some of the great eras of the past.* Michael Atiyah

As ideas get tested, the validity of some become more and more generally accepted and expanded upon. Whether 'pure' and very abstact, or closely related to application, mathematics and all the sciences provide an environment for the scrutiny of ideas, and insight into phenomena and problems which ideas try to describe or solve.

3.4 Research continues

> *What a piece of work is man! How noble in reason. How infinite in faculty.* William Shakespeare

The continued pursuit of better understanding demands the continuation of research. While technological and applied research are more easily motivated, there is also need for investigations which may not have immediate application.

It is precisely the demand for better understanding, and a desire to *see into the light of things,* that supports research. To deny fundamental or pure (unapplied) research is to suppose that what is already known is unconditionally sufficient, and ultimately implies the acceptance of ignorance and limitation. Fortunately, this is not only philosophically unsound, but also ignores the unanswered questions encountered in mathematics and science today.

The rewards of continued investigation are not obvious. Research demands the development of imaginative, critical and analytical thought. Such skills developed by someone pursuing more pure (unapplied) or abstract research are easily transferable to more applications related problems. Furthermore, it is easier to train students with more general skills, and to concentrate on fundamental phenomena, than to teach specific skills that are also likely to be rendered obsolete with rapid technological change.

The direct relevance or application of pure or fundamental research is more obscure. While the turnover in commercial or technological research may not be immediate, the turnover in pure or fundamental research is much slower. It is also accepted that what is esoteric at one stage can become widely known or applied at another. Most discoveries eventually find application. We may repeat the well-known the tale of G.W. Hardy's claim that his work in number theory and convex analysis were useless and would never have any use. Ahead of their time as theories for practical purpose, complex analysis is now used extensively in engineering, while number theory is fundamental to cryptography. Alternatively we may recall how centuries of scepticism, and seemingly dubious investigation, eventually gave rise to non-Euclidean and Riemannian geometries, which, in turn, eventually led to the historically significant theory of general relativity attributed to Einstein. Thus, not only does continued research clarify existing ideas, it also paves the ways for new insight. Insight continues to be the result of years, and sometimes centuries, of patient enquiry and investigation, with direct and indirect benefits enjoyed in the short and longer term.

> *Pure mathematics goes back (at least) to about 1600 B.C. Applied mathematics is, of course, older.* Kleiner and Movshovitz-Hadar

We thus see that these subjects involve a cumulative tradition of knowledge. In medicine for example, traditional remedies were discovered over long (unwritten about) periods, and passed down from sages to their apprentices. It is continued investigation that strips traditional knowledge of its inaccuracies, myth, superstition and mystery. Mathematics and science involve histories of ideas which are the products of time, people and trends in thought. Unfortunately greater society is still misled by tales of miraculous discovery by unique genius. While Einstein stands as a useful icon of scientific endeavour, he did not work in isolation, nor were the new theories he expounded upon independent of older mathematical ones. It was continued investigation, not isolated consideration that led Newton and Leibnitz to develop the calculus. In fact the ideas for the calculus had their origins in Greek mathematics.

> *If I have seen further, it is because I have stood on the shoulders of giants.* Isaac Newton

Modern math has thousands of contributors spread over the globe. We can now remark how the classification of finite groups, mathematical structures in modern algebra, was a collaborative effort of dozens of researchers who published thousands of journal pages of results on work spanning several decades.

Unfortunately, mathematical history and its contributors are unknown to the broader public. One does not want to propagate the myth of single genius. However, the telling of life-stories of mathematicians, and the times they lived in, can show mathematics as a human endeavour.

> *From the infinitesimal calculus to the present, it seems to me, the essential progress in mathematics has resulted from successively annexing notions which, for the Greeks or the Renaissance geometers or the predecessors of Riemann, were* outside mathematics *because it was impossible to define them.* Jacques Hadamard

As we continue to enquire and try to understand, we 'discover' new ideas which may have been inaccessible to our understanding previously.We also refine and expand on ideas which we thought were already understood. Thus, we are able to have deeper and wider insight into the greater order of things.

3.5 Mathematics and science continue to play a role in society

Knowledge is power. Francis Bacon, 17th century

We may be inspired by the beauty and scope of mathematical and scientific theories and activity. Mathematics and science have also been driven by application and technology which serve to improve the way we live. It must be noted, though, that these subjects are not just about the improvement of techniques. New insights can mean the development of new techniques or application. We have already indicated some ways in which mathematics and science are intimately and irremovable connected to the advance of civilisation by application.

We also need to be aware of the dangers. The use of mathematics and science requires responsibility as well. Mathematical solutions to practical problems 'have to get it right' when there are lives and human condition at stake - bridges must not fall down, electronically controlled train systems must work to prevent accidents, and economic models must provide for the economic needs of the people involved. Sometimes the scope of application demands ethical debate. The use of mathematics and science to build weapons of destruction such as the atom bomb poses interesting and very important questions about the role of science. Scientific ignorance gave humanity (disproved) theories of racial superiority. Moreover, such theories were used to substantiate social engineering and economic policy which continue to have deep social, cultural and economic consequence. Scientific ignorance and abuse have also lead to the abuse of our environment and natural resources. Thus, science also requires an honesty of where we have made mistakes. Insisting on adhering to a flawed mathematical model, or and incorrect medical theory claiming cure, can be disastrous.

Fortunately, the solutions to some of humanities problems can be found in the enlightened use of mathematics and science. Mathematics and science give us some measure of control over ourselves, society and the environment. The impact of application, must therefore be appreciated and understood - we are all affected by our collective knowledge and ignorance.

The development of mathematical and scientific awareness and education therefore are critical for our collective advance. Faced with the vastness of scope, it is impossible to provide a definitive collection of skills in any given discipline or branch of science. What is possible is the transference of the understanding of basic principles on which insight can be built. With this, one can foster the courage and willingness to look at things in new ways and to tackle problems.

All our science, measured against reality, is primitive and childlike - and yet it is the most precious thing we have. Albert Einstein

References

[1] M.Kac, G.Rota and J.Schwartz, *Discrete Thoughts*, Birkhauser, Boston, 1986.

[2] J.Goldemberg, *What is the Role of Science in Developing Countries* SCIENCE 279, Febrauary 20, 1998; pp 1140 -

[3] Carl Sagan and Ann Dryuan, "The Path to Freedom" in Carl Sagan's *The Demon Haunted World*, Headline Book Publishing, 1997, Chapter 21.

[4] Kenneth Manning, *Science and Opportunity* SCIENCE 282, November 6, 1998, pp 1037 -

[5] Joseph Stiglitz, *Public Policy for a Knowledge Economy*, available online at URL http://www.worldbank.org/html/extdr/extme/jssp012799a.htm.

[6] Eduardo Martinez, *Boosting public understanding of science and technology in developing countries*, available online at URL http://helix.nature.com/wcs/c16.html.

[7] Claude Lobry, *Mathematics and Development*, available online at URL http://helix.nature.com/wcs/c17.html.

[8] Isaac B. Tabata, *Education for Barbarism*, Pall Mall, UK, 1960.

[9] MANY MORE *more* more, 2000.